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ABSOLUTE AND CONVECTIVE INSTABILITIES OF THE HELIOPAUSE

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Abstract Stability of shear flows is of fundamental importance in solar and solar-terrestrial physics. Examples of such flows include plasma flows, e.g., in the vicinity of the magnetopause of the Earth or planets, the boundaries between fast and slow streams of the solar wind or the flow in the vicinity of the heliopause. The normal mode analysis is not sufficient to predict if a finite portion of a shear flow looks stable or unstable. The reason is that this analysis deals with spatially periodic perturbations, while real perturbations are always confined to a finite region. To study the stability of a shear flow with respect to perturbations finite in space we have to solve an initial-value problem. Then two scenarios are possible. In the first scenario the initial finite perturbation exponentially grows at any spatial position. Such a type of instability is called absolute. In the second scenario the initial perturbation also grows exponentially, but it is swept away by the flow from any finite region so fast that it decays at any fixed spatial position. Such a type of instability is called convective. The classification of absolute and convective instability is important for the understanding of the physical processes in solar, solar-terrestrial and astrophysical plasmas, and for the interpretation of in-situ observational data like STEREO.

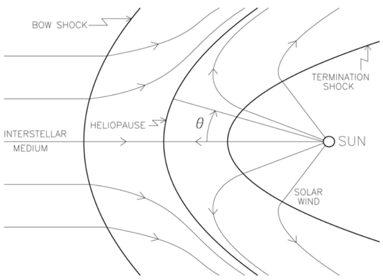

Fig. 1: The model.

1 Introduction

Our motivations for the present study is to analyse the instabilities of the near flanks of the heliopause in the model of the solar wind -- interstellar medium interaction (*Fig. 1*) first suggested by Baranov et al. 1971. The dynamics of small localized disturbances is investigated in a KH-type flow in which one of the fluids is inviscid, but the other one is viscous, and no surface tension is present on the interface. A zoom of the simplified flank region is shown by *Fig. 2*.

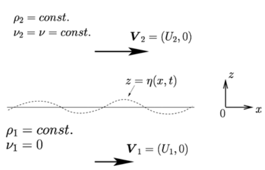

Fig. 2: The flank.

For modelling the heliopause in the framework of the stability analysis, we suppose that the linear perturbations considered possess the characteristic wavelength which is much smaller than the curvature radius of the heliopause at the apex point. Then a near flank of the heliopause can be assumed to be a planar tangential discontinuity and a local quasi-parallel stability analysis applied (*Fig. 2*). In this approach, the flank of the heliopause is a plane, and the base plasma flow on both sides of the flank is treated as being open, space-independent, unidirectional and parallel to this plane. Restricting our consideration to relatively small polar angles (θ<30°), where the plasma flow on both sides of the heliopause is strongly subsonic, we can use the incompressible fluid approximation. The plasma on both sides of the heliopause is a rarefied gas, and, hence, effectively no surface tension is present on the heliopause.

2 Solution to the boundary- and initial value problem

The perturbation interface $\eta(x,t)$ can be formally expressed as an inverse Laplace-Fourier integral given by

$$\eta(x,t) = \frac{1}{4\pi^2}\int_{i\sigma-\infty}^{i\sigma+\infty} e^{-i\omega t}\left[\int_{-\infty}^{\infty}\frac{T(k,\omega)}{D(k,\omega)}e^{ikx}\,dk\right]d\omega.$$

Here the dispersion function, *D(ω,k)*, represents the model, whereas the function *T(ω,k)* depends on the initial and external perturbations. For studying absolute and convective instabilities of, and signalling in, the model, it is sufficient to treat the asymptotics of the perturbation interface given above and show that the roots of *T(ω,k)* do not cancel the corresponding contributions.

3 Normal modes are monochromatic disturbances satisfying the dispersion relation *D(ω,k)=0*. Kikina (1967) showed that for any non-zero value of real *k* there exists one and only one unstable normal mode and the growth rate is uniformly bounded ⇒ the initial-value problem for localised disturbances is well-posed!

4 Absolute and convective instability

To distinguish between the absolute and convective instability we have to study the asymtotic behaviour of $\eta(x,t)$ at a fixed x as $t\to\infty$. This analysis has been done with the use of Brigg's method (Briggs, 1964). For equilibrium values from Baranov et al. (1979) in the interval 10º≤ θ ≤30º **we found all the instabilities are convective**. These results are in excellent agreement with the results of numerical studies by Belov & Myasnikov (1999).

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